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How proportional are electoral systems? A universal measure of electoral rules☆

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ABSTRACT

The proportionality principle of electoral systems plays an important role in multiple literatures, including comparative politics, political economy, and conflict studies. The single most important electoral system feature used to determine whether an electoral system might provide for proportional outcomes is district magnitude, i.e., the number of representatives that are elected from a given district. However, once we move beyond “simple” systems, measuring the institutional side of the equation becomes more complex. The legislative electoral systems of three out of five (61%) contemporary democracies rely on multiple institutional rules that affect the proportionality of their seat allocation, for example by combining different district magnitudes in multiple tiers, or delimiting the number of parties that can win seats through legal thresholds. There is no widely accepted measure of district magnitude for such cases. Relying on four simple axioms, this article extends the concept of district magnitude to all these complex electoral systems. We introduce the effective district magnitude, a new index that measures the proportionality of electoral systems based solely on institutional rules. It is applicable for all rules used in national parliamentary elections in democracies worldwide. This new, theory-based index improves the explanatory power of two crucial variables usually associated with district magnitude: vote-seat disproportionality and party system fragmentation.

1. Introduction

In the literature on democratic theory, understanding the degree to which voter preferences are represented in parliament is almost always defined in terms of some form of the *proportionality principle*. According to this principle, groups of voters should be represented in rough proportion to their numbers (Mill, 1861). Most often, the principle is studied in terms of how votes expressed for political parties in elections are translated into parliamentary seats (Rae, 1967; Taagepera and Shugart, 1989).

Understanding the proportionality of an electoral system has important implications for many aspects of politics. The proportional properties of an electoral system have been shown to be relevant variables in comparative work across many areas of political science: political economy (e.g. Persson and Tabellini, 2000, p. 187–201, 2003, p. 81–112; Iversen and Soskice, 2006), the literature on civil conflicts (Saideman et al., 2002; Alonso and Ruiz-Rufino, 2007; Schneider and Wiesehomeier, 2008), and comparative politics (Duverger, 1951; Lijphart, 1999, p. 143–179), e.g. for the stability of government coalitions (Taagepera and Sikk, 2010), the responsiveness of politics to changes in public opinion (Powell, 2000; Portmann et al., 2012), and political behaviour (Hix et al., 2017).[1]

The question as to whether given electoral systems offer political parties, regardless of their size, equal access to parliament, has been assessed either by analysing electoral outcomes, or by studying electoral rules. Scholars are primarily interested in whether multiple political parties can gain seats in parliament in proportion to their vote share. Accordingly, we rely on two main outcome-based measures, one for the

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[1] Findings of some of these studies are highly sensitive to the measure of proportionality (Höhmann and Tober, 2018).

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direct effect of proportional electoral rules, measuring discrepancies (disproportionalities) between the distribution of votes and seats, and one for their joint direct and indirect (anticipatory) effect onto the formation of parties, the effective number of parties in parliaments. These can be established *'a posteriori'*, for each election, and are explained by several *'a priori'* features of electoral rules on the institutional side.

The classical distinction was between *plurality* (first-past the-post) or majoritarian electoral systems and proportional representation (PR) (Duverger, 1951). Recognising that proportionality is gradual rather than dichotomous, Rae (1967) identified district magnitude, defined as the number of seats elected in a constituency, as the main continuous institutional feature of an electoral system, to determine whether a system is proportional. While this measure holds well for simple, district-based electoral systems, contemporary democracies increasingly rely on additional rules and intermediate solutions between PR and plurality/majority, which render the picture more complex. As of 1990, 61% of democracies worldwide have used one or more elements that move beyond the classical distinction between PR and majoritarian rule, or complement district magnitude with other limits to party representation, in at least one election (Bochsler and Bernauer, 2014). Dozens of countries have introduced mixed electoral systems or other multi-tier electoral systems, combining seats elected according to different sets of rules, while others require parties to gain a certain share of national votes to qualify for the allocation of seats ('legal thresholds'). Yet others allocate special bonus seats to the largest party or parties. Thus, to assess the even (proportional) access of parties to parliament, district magnitude is no longer the only relevant important institutional variable; its effect on party representation can only be understood in the context of further rules.

This paper is interested in finding a single score that best describes the variety of the world's democratic electoral systems in terms of their institutional propensity to allow political parties of different sizes to gain representation in proportion to their vote share.

We offer a new comprehensive and simple metric of this proportionality principle. Our new index is based on theoretical grounds, entirely institutions-based (a priori), without considering electoral results (*a posteriori*). It modifies the classical measure to assess the institutional rules of electoral systems, district magnitude, *m*, in a way that allows for a universal characterisation of the five highlighted complexities in contemporary electoral rules. District magnitude is defined as the number of seats elected in a constituency. We develop a theoretical framework that allows the assessment of electoral systems with multiple, non-correspondent layers of constituencies, or with legal electoral thresholds, with regards to their institutional opportunities for proportional representation. Thereby, we consider the main electoral system features that are commonly seen as affecting the proportionality of the seat allocation.[2] Our index allows scholars to compare the impact of different elements of electoral systems. Inter alia, it offers a framework to assess the consequences of electoral reforms (Lago and Martinez i Coma, 2021).

Our new approach builds on four theoretically grounded axioms that allow us to assign proportionality scores to five complex elements of electoral systems. This results in a simple and single index, which we label the 'effective district magnitude'. Some of the suggested partial solutions build on earlier work by Grofman (1975), Sartori (1994, p. 18–19) and Lijphart (1994). Others are entirely new, and our way of combining them is innovative. This approach is modular, yet we apply all four rules jointly, because complex electoral rules do not manifest in isolation, but are instead embedded in the electoral system. The entirety of these rules has a joint impact on the actors' behaviour and on party representation in parliament.[3]

We validate the 'effective district magnitude' empirically. We do so by comparing our metric's explanatory power against earlier operationalisations of district magnitude. We demonstrate that 'effective district magnitude' allows us to improve the estimations of models explaining outcome-based measures related to proportional representation, i.e. vote-seat disproportionality, and the effective number of parties in parliament. The latter measures the effect of the electoral system on party concentration, a direct consequence of the proportionality profile of a particular electoral system. We do this for a dataset of 530 elections in 94 democracies, for which we have combined and complemented the information from several existing projects.

In the next section, we discuss earlier approaches used to measure the proportionality principle in electoral systems; thereafter, we identify problems in appropriately specifying district magnitude, show how our approach solves those problems, and contrast our approach with that of previous research. Section 4 assesses the empirical validity of our approach.

2. Literature review

The main concern in the debate about electoral systems is their degree of inclusiveness and proportionality in representing societies. Mill stated that in a democracy, any segment of society should be represented proportionally, so that "a majority of the electors would always have a majority of the representatives; but a minority of the electors would always have a minority of the representatives" (Mill, 1861, p. 133).

The literature on electoral systems and party systems is not only interested in whether political parties gain representation in parliament, but whether parties can obtain a fair share of the seats in parliament in proportion to their votes. For instance, in plurality or majority-vote systems, smaller parties often win individual seats in their local strongholds (Grofman, 2001; Taagepera, 2002), but they usually remain marginal (Kendall and Stuart, 1950). Accordingly, the most prominent indices that assess political pluralism and the proportionality principle in election outcomes, *a posteriori,* do not count the absolute number of parties in parliament (i.e. *whether* parties gain seats), but instead take the sizes of parliamentary parties into account (Laakso and Taagepera, 1979), and compare them to the respective parties' vote shares (disproportionality scores (e.g. Loosemore and Hanby, 1971; Gallagher, 1991; Taagepera, 2007, p. 179–183)). It turns out that some of the countries with the most majoritarian electoral systems, such as the United States, are also characterised by low disproportionality. The reason for this is that there are only two large parties, with smaller parties being pushed out of the competition due to their almost nonexistent chances of winning seats. *A posteriori* measures vary among countries with similar electoral rules, and from election to election, as they reflect not only the set rules on elections, but also the social cleavages in place, or the strategic behaviour of parties and voters. Therefore, such *a posteriori* measures also have limited validity as measures of political institutions.

Scholars investigating the opportunities offered by electoral systems for (proportional) party representation address the (a priori) institutional side of the equation as electoral system "permissiveness" (Clark and Golder, 2006, p. 683; Taagepera and Shugart, 2017, p. 262; Riera, 2015) or "openness" (Taagepera, 2007, p. 83). The most important institutional feature related to the proportionality principle is the

[2] This has been addressed as the "mechanical effect" of electoral systems. Vice-versa, behavioural (also "psychological") effects, not covered in this paper, are about the choices by voters and parties.

[3] E.g., some electoral systems function almost like perfect PR (with compensatory proportional seats in upper tiers), but with legal thresholds embedded, so that small parties are excluded from the proportional seat allocation part.

district magnitude *m*, the number of representatives elected from single districts.[4] It is part of Powell's "proportional dimension" (2000).

There are different ways to operationalize the institutional features related to this proportional dimension of electoral systems. Some scholars rely on categorical measures, where mixed or complex systems are classified as intermediates between PR and majority/plurality systems (Reynolds et al., 2005, p. 55–56; Kostadinova, 2002; Lindberg, 2005; Best and Seyis, 2021). Others add binary variables for some elements of electoral systems (legal thresholds, multiple tiers) (Carey and Hix, 2011), or apply multi-variable solutions for various elements (Amorim Neto and Cox, 1997; Lublin, 2017). We instead suggest to calculate a universal, continuous measure of district magnitude which covers the most important complexities in contemporary electoral systems in a single variable.

3. Four axioms to extend the scope of the concept of district magnitude to deal with five complexities in electoral systems

In this section, we discuss five complex features of electoral systems, and ways to deal with them. These are PR seat allocation formulas, legal thresholds, compensatory and non-compensatory mixed and multi-tier electoral systems, as well as PR rules with bonus seats. This section highlights the problems related to measuring district magnitude in these systems, and introduces new solutions. This is based on four axioms: one that applies to all problems, and three axioms that highlight specific issues related to some of the five complex elements of electoral systems. From these axioms, we develop four rules that allow us to extend the measure of district magnitude to all lower-house electoral systems of contemporary democracies, worldwide.

Axiom 1. (universality): Our first axiom corresponds to the goal of the paper: the metric we use to develop a unidimensional characterization of an electoral rule's relationship to the proportionality principle should provide a single and meaningful score which is comparable in its implication across electoral systems. It should depend upon institutional features (electoral formulas, numbers of seats, district magnitudes, and legal thresholds, across all relevant tiers), if and when these features affect the possibility of parties to win seats in relation to their votes. Meaningful scores on this metric define a finite scale that is comparable to district magnitude in simple electoral systems.

Our axioms are translated into four rules. The first rule deals with different seat allocation rules. Rules 2–4 deal with situations where an electoral system consists of multiple constraints. Such constraints can either apply sequentially, where each constraint constitutes a necessary condition (legal thresholds, cf. rule 2), they can apply in parallel, for two different parts of parliament (as exemplified by MMM rules or systems with bonus seats, cf. rule 3), or they can apply conditionally, whereby passing one constraint suspends the need to pass the other constraint (for instance for MMP rules, cf. rule 4). For each rule, the subsequent subsections offer a short explanation. While the main text offers guidance to authors on how to proceed when applying our approach, along with calculation examples, we refer to appendix A1 for a more detailed discussion of the axioms and formal notations of the rules.

3.1. Complexity 1: different PR seat allocation formulas

3.1.1. The problems

For some of the district-based electoral systems, district magnitude *m* is not their best characterisation with regards to the proportionality principle. This problem has been recognised in the literature for the multi-seat plurality or majority vote (Taagepera, 2007, p. 178–179).[5] However, there are also subtle differences between the most common D'Hondt seat allocation formula and alternative PR seat allocation formulas: under the D'Hondt rule, the 'threshold of exclusion', i.e. the vote share a party needs to win its first seat in parliament, is determined by district magnitude in accord with the function $T_E = 1/(m+1)$. For instance, a 5-seat district by D'Hondt leads to a threshold of 16.7%. The same applies for the Single Transferable Vote and the Single Non-Transferable Vote (Grofman, 2001). Other PR formulas, including Sainte-Laguë, are more permissive: a 5-seat district would be associated with an entry threshold of 10% of the votes ($T_E = 1/2$ m) (Grofman, 1975).[6]

3.1.2. Rule 1

Our unified approach, which involves assigning an adjusted score of district magnitude both to multi-seat PR systems, as well as to PR seat allocation formulas other than D'Hondt, relies on the Threshold of Exclusion. Any electoral district which has the same Threshold of Exclusion as a PR district with a D'Hondt seat allocation rule, will have the same score on our new adjusted district magnitude metric. Thus, for instance a 3-seat district has the same threshold as a 5-seat-district with D'Hondt, or more generally, $m_{eff} = 2m\text{-}1$.

We proceed by transforming the district magnitude into $m_{eff} = 2m\text{-}1$ for all elections (or for tiers of multi-tier election systems) with Sainte-Laguë and divisor-based seat allocation rules. Further, elections in multi-seat plurality or majority districts are always associated with a threshold of 50%, and with $m_{eff} = 1$.

3.2. Complexity 2: legal thresholds

The metric we use to develop a unidimensional characterization of an electoral rule's relationship to the proportionality principle should incorporate constraints such as legal thresholds. Legal electoral thresholds now apply to one in two elections to a national parliament worldwide. In elections with legal thresholds, only political parties that reach a defined vote share are allowed to take seats in parliament, potentially denying many parties representation, even in cases where district magnitude would allow them to win seats. Sometimes the district magnitude is more constraining than the national legal threshold, sometimes the reverse is true.

Axiom 2. When there are two or more constraints, and parties only win seats if they fulfill both constraints (serial conditions), the feature that is the more constraining is the one we should use to calculate the metric.

3.2.1. Rule 2

We convert legal threshold into equivalent D'Hondt values of m via the *Threshold of Exclusion* (cf. Lublin, 2017). For example, a 5% legal threshold is equivalent to a 19-seat district with D'Hondt.[7] For any given

[4] Majoritarian rules with multi-member districts are usually considered as m = 1, they are as restrictive as single-member districts, if not more restrictive.

[5] However, as Benoit (2001) notes, they are absent from most samples of national parliament elections.

[6] This 'threshold of exclusion' applies under Sainte-Laguë if all other votes are concentrated on a single party. If taking into consideration the number of parties running in election, a different solution might be possible: when votes are evenly distributed among *m* parties, the D'Hondt threshold of exclusion (*1/(m + 1)*) applies to all PR formulas (Lijphart and Gibberd, 1977). According to our 'a priori' approach, we do not consider '*a posteriori*' information, such as the number of parties or vote distributions. Scholars who decide to follow the 'm equal parties' assumption may still use our index, and disregard rule 1 (as shown in appendix D1 and D3).

[7] This is similar to Lijphart's (1994) "effective threshold", an intermediate solution between the *Threshold of Representation* and the *Threshold of Exclusion*. Here we only use the *Threshold of Exclusion*. Very few articles on national effects of electoral rules "drill down" to the local level to consider this issue.

district, whichever value of m is the more stringent of the two constraints is the one we use, or $m_{eff} = \min\left(m, \frac{1}{t-1}\right)$.

3.3. Complexity 3: mixed and multi-tier electoral systems with no compensatory features

3.3.1. The problem

As of the 1990s, the number of democracies electing their parliament by mixed or multi-tier electoral systems has grown, rendering it important to identify a good way to measure likely proportionality effects of such rules (Elklit and Roberts, 1996; Massicotte and Blais, 1999). However, it is not at all clear how best to combine information about the two tiers, and the previous literature has offered a diverse set of answers.

Instead of combining information about the two tiers into a single score, some scholars assign two different scores for each part of a mixed system. This works well for the analysis of the different effects of each tier e.g. on patterns of party competition (Moser, 1999).[8] However, many other research questions require a single score. This is also done in recognition of the fact that all tiers are part of the same system and the parties competing in the different tiers are the same, the elected MPs sit in the same parliament, and citizens notice whether their preferences are represented in parliament or not, and less so in which tier. Some lawmakers even explicitly design mixed electoral systems with multiple components to balance the properties of two different logics of representation. In light of this, we aim for a central measure that considers the properties of each of the multiple tiers, according to their importance in the system.

Axiom 3. When there are multiple modes (and constraints) according to which parties can win seats in parliament, and each mode applies for a different category of seats in parliament (parallel constraints), then (a) the measure of effective district magnitude should be affected by each of these modes. When both (all) tiers are equally large, then (b) a marginal change of effective district magnitude in any of the tiers should have an equal marginal effect on our score. The rules (district magnitude) that apply in the larger tier should be (c) given greater weight than those applying to the smaller tier; and an increase in the size of one tier should (d) increase its weight and converge towards the district magnitude of the larger tier if it becomes dominant.

3.3.2. Earlier work

Other authors who have aimed at a single score rely on the electoral formula and district magnitude of the lower tier, which supposedly is the more important one (Clark and Golder, 2006; Golder, 2006, p. 37),[9] and possibly add a second variable, counting the number of upper tier seats (Amorim Neto and Cox, 1997; Clark and Golder, 2006; Taagepera and Shugart, 2017, p. 262–267; Lublin, 2017). A second group of authors identify a central tendency that relies on the rules in both tiers, such as the magnitude of the district where the median legislator is elected (Cox, 1997, p. 208–209). Renwick and Pilet (2016, p. 35–36) instead take the arithmetic mean of the district magnitudes across tiers. However, recognising that in many mixed systems one tier is considerably larger than the other, they weigh their mean by the number of seats in each tier (calculation example in Table 1). In an alternative approach, Milesi-Ferretti et al. (2002) consider *m* only in the (most) proportional of the tiers, as this tier determines the *Threshold of Exclusion*. In most cases, this implies that they neglect the lower tier, because it is more restrictive. Each of these solutions violates some aspects of our axioms 1 and 3. A frequent violation is that the rules in one of the two tiers do not affect the solution; other solutions suffer from non-monotonicity, or they result in extremely high scores (see also Table 1). Appendix A2 offers a systematic review of the axiomatic properties of the approaches listed here.

3.3.3. Rule 3 and comparison to other approaches

We illustrate calculations of our metric for the MMM systems used in Mexico, Lithuania and Thailand in Table 1.[10] First, we calculate (effective) district magnitude for each tier separately, according to the previous rules. In particular, multi-tier systems may have legal thresholds in one tier but not the other, and we calculate the tier-specific district magnitude (m_{LOW}, m_{UP}), relying on rules 1 and 2, for the tier (s) where thresholds apply. In a second stage, we weigh district magnitude in each tier by the tier's size (S_{LOW}, S_{UP}), and calculate the weighted geometric mean thereof, which is defined as $m_{eff} = m_{eff,LOW}^{\left(\frac{S_{LOW}}{S}\right)} \cdot m_{eff,UP}^{\left(\frac{S_{UP}}{S}\right)}$.

In the three countries in our calculation example, the share of upper tier seats varies from nearly 50% (Lithuania) to 25% (Thailand), and district magnitude varies between 71 seats (Lithuania) and 375 (Thailand). Lower-tier district magnitude is equal to 1, in all MMM systems. This means that we multiply the lower-tier and the upper-tier magnitude, each raised to the power of the share of seats allocated in the respective tier, e.g. in the Lithuanian case $m_{eff} = 1^{\frac{71}{141}} \bullet 70^{\frac{70}{141}}$. This calculation assigns the highest score to Lithuania, followed by Mexico and Thailand, in line with the shares of upper-tier seats in each.

If we look at the other approaches reported in Table 1, we can observe some of their axiomatic properties. Because they focus on lower-tier magnitude, Clark and Golder assign a score of 1 to any MMM systems (while Cox does so for most MMM systems). Taagepera and Shugart's extended magnitude-seat product assigns a high score to Mexico, because its rather large upper tier coincides with a large absolute number of lower-tier seats. Again, the upper-tier rule does not enter into the calculation. Renwick and Pilet rank Lithuania and Mexico in the same order as we do, though they rate Thailand as much more proportional due to the large district magnitude in the upper tier, and their reliance on the arithmetic, rather than the geometric mean (see appendix A2, Axiom 3d). We rate Thailand as less proportional than the other two exemplary cases, due to its relatively small upper tier. Also, Renwick and Pilet's scores for MMM systems are 4–9 times larger than ours. Thus, our approach to tiered systems without compensatory seats is clearly distinct from those in the previous literature.

3.4. Complexity 4: mixed and multi-tiered systems with compensatory seats

3.4.1. The problem

MMP systems, and other multi-tier electoral systems with a compensatory PR rule, have very different properties with regards to proportionality than non-compensatory or MMM systems. In mixed-member proportional (MMP) systems, the compensatory upper tier functions similar to a PR system that applies for *all* seats in parliament, whereby the seats gained in the lower tier are counted towards the number of seats that a party is allocated. Some authors treat MMP systems as sub-types of proportional representation, complemented by local districts to allow for personal and territorial representation (Kreuzer, 2004). However, most earlier work (Amorim Neto and Cox, 1997; Taagepera and Shugart, 2017, p. 299–302; Clark and Golder,

[8] Much of this work demonstrates how institutional effects can spill over across tiers (Herron and Nishikawa, 2001; Cox and Schoppa, 2002) so, even though each tier is viewed separately, the results in individual tiers are not independent of the results in the others.

[9] This recommendation is empirically based (Clark and Golder, 2006, p. 693, 699).

[10] To simplify the calculation, table 1 does not consider legal thresholds, or the type of PR formulas that apply for the upper tier seat allocation by PR. Appendix A1,table A1provides the corrected values. For instance, for Lithuania, in consideration of the 5% legal threshold, and in application of rule 2, we calculate an effective district magnitude of 19 seats ($m_{eff} = 1/t-1$), rather than 70. This leads to an overall effective district size of 4.3 instead of 8.2.

Table 1
MMM - Calculation examples of overall district magnitude, disregarding legal thresholds.

Example	Electoral system properties		Resulting score			
	lower tier (plurality or majority)	upper tier (proportional)	Clark & Golder (lower tier m); Cox (median district)[a]	Taagepera-Shugart (lower-tier mS product and upper-tier correction[b])	Renwick-Pilet (weighted arithmetic mean)	Weighted geometric mean (disregarding legal thresholds)
Lithuania	m = 1, S = 71	m = 70, S = 70	1	1088	35.3	8.2
Mexico	m = 1, S = 300	m = 40, S = 200	1	2705	16.6	4.4
Thailand, 2011	m = 1, S = 375	m = 125, S = 125	1	1482	32.0	3.3

[a] Amorim Neto and Cox complement their measure, which is based on the lower-tier rule, with a second variable for the share of upper-tier mandates.
[b] Taagepera and Shugart (2017, p. 263) do not report a corrected seat-magnitude product, but use their score to predict the number of parties. The corrected product can be calculated backwards, complemented by the upper-tier seat share t: $mS_{corr} = ENP_{pred}^{6} = 2.5^{(t\bullet 6)}\bullet mS$.

Table 2
MMP - Calculation example with threshold in upper tier, Bolivia.

Example	district magnitude	legal threshold	effective district magnitude	# seats
lower tier	1	–	1	68
upper tier	62/9 = 6.9	3%	6.9	62
overall		3%	(62 + 68)/9 = 14.4	130

2006; Cox, 1997; Renwick and Pilet, 2016) treats MMM and MMP systems as equivalents when it comes to measures of district magnitude.

Axiom 4. Some electoral systems have parallel constraints, i.e. a party will gain representation even if it only passes one of several barriers for entry. For instance, MMP systems allocate seats either to parties that win seats in the lower tier (district tier), or to parties that gain seats in the upper tier, whereby parties gaining seats according to upper tier rules participate in the seat allocation at large. In such cases, the effective district magnitude is determined by the lowest of all constraints, under the condition that this lowest constraint allows parties to gain a proportional share of all seats in parliament.

3.4.2. Rule 4

For MMP or multi-tier PR rules with compensation, we base our calculation on the number of seats in parliament (or the number of seats in all linked tiers), and the number of districts (if any) in the upper (compensatory) tier. We consider this system to be equivalent to a PR system based on the upper tier rules. Hence, we calculate effective district magnitude from these characteristics. If a national legal threshold applies in the compensatory tier (rule 2), we consider it at this stage.

Table 2 offers a calculation example for Bolivia, where the upper tier of 62 seats is divided into nine electoral regions, and each region is complemented with lower-tier seats. For MMP systems with no upper-tier districts, the effective district magnitude corresponds to the overall number of seats in parliament. When the upper tier rule includes legal thresholds or a special PR formula, solution rules 1 and 2 apply.

$$m_{eff} = min\left[\frac{S_{LOW} + S_{UP}}{d_{UP}}, \frac{1}{t} - 1\right]$$

In some elections, the number of upper-tier seats is small, and there are not enough seats to allow an allocation that satisfies the upper-tier PR rules. A clear-cut case thereof is Bulgaria, with only 1 upper-tier mandate in each upper-tier district. In this situation, *effective district magnitude* should approach lower-tier district magnitude. In a handful of other elections, including elections in Albania and Venezuela, there are 29% or 50 % upper-tier districts, respectively, which allows the disparities in representation between small and large parties to be somewhat reduced, but due to the low thresholds, these might not be sufficient to provide for proportional outcomes. Appendix A4, relying on Bochsler (2023), offers a formal solution to extend our approach. It identifies MMP systems for which the seat allocation is entirely reliant on the upper-tier rules (i.e. which function similar to PR systems), and others with a small number of upper-tier seats, which are expected not to be entirely proportional. This extension requires a set of assumptions about the vote distribution in elections, adds a layer of complexity, and it turns out that the improvement of the empirical estimations on our dataset are minimal (appendix C2). Therefore, we do not incorporate this nuanced solution in our measure of 'effective district magnitude'.

There are several special cases that require consideration. These cases are sufficiently rare to relegate the technical details to Appendix A3. The first case consists of MMP systems (Romania in 2008) or multi-tier compensatory systems (Austria), which do not assign a fixed number of seats to the upper tier, but instead have flexible tier sizes. Second, the Philippines applies a unique cap to the number of upper-tier mandates, not allowing any party list to gain more than 3 upper-tier mandates. Third, Hungary elected its parliament according to a three-tier system until 2010. In multi-tier PR systems with more than one upper tier, we apply our rules about multi-tier systems sequentially for each tier.

3.5. Fifth complexity: systems with bonus rules

3.5.1. The problem

The fifth complexity regards PR systems with bonus seats, where, on top of its proportionally allocated seats, the largest party is awarded a certain number of additional seats. Some articles (e.g. Li and Shugart, 2016) exclude such cases from their analysis, others treat them as ordinary PR (Clark and Golder, 2006).

3.5.2. Our solution for bonus rules

We do not need a special solution for bonus seats; instead, we treat them as analogous to MMM electoral systems, which consist of a lower tier elected by PR, and an upper tier elected by plurality (multi-seat plurality district with blocked party lists, m = 1).[11]

[11] We thus associate the upper tier of PR-bonus systems with the highest possible effective threshold (50%) and the lowest possible effective magnitude ($m_{UP} = 1$) possible according to our axiom 1. Arguably, the bonus tier (similar to the block vote) might have a stronger effect of party concentration than a lower tier with the same number of seats, and single-seat districts (see also analysis of residuals in appendix C5). Instead of providing an ad-hoc solution to this problem, we relegate this to work that takes into consideration the number of districts.

3.6. Other elements of electoral rules

Recent studies have shown that uneven district magnitude within single-tier systems can affect the proportionality profile of an electoral system. More specifically, districted PR systems with uneven district magnitude are more permissive (Kedar et al., 2021). Our axiom 3 can be applied to cases of uneven district magnitude. In appendix A5, we introduce a solution that is analogous to the weighted geometric mean of district magnitude that we rely on for MMM systems.[12] For electoral systems that combine PR districts with isolated seats elected by the plurality/majority vote in the same tier (Italy after 2006, Panama, Switzerland), we implement the solution in our main analysis.

While our extension of district magnitude allows us to characterize most of the world's electoral rules with respect to *effective district magnitude*, we do not address: (1) the different implications of plurality versus runoff methods (Grofman, 2008), (2) list apparentments for PR systems (Bochsler, 2010), and (3) special rules for social minorities, such as quotas. Other elements of electoral systems (e.g. the ballot structure, with options such as cumulative or panachage voting) are related to voting behaviour and to the intra-party choice. However, they appear to a lesser degree to be connected to the mechanical effect of electoral systems.

Also, we restrict this approach to ex-ante institutional features of electoral systems, and therefore do not consider '*a posteriori*' (outcome-based) features, such as the number of electoral competitors, which might interplay with the institutional rules (see subsection 3.1 for an example).

4. The empirical performance of our new measure of *effective district magnitude*

This section offers criterion and construct validity tests of our new theoretically based indicator, *effective district magnitude*.[13] We therefore correlate it with widely used variables that are *a posteriori* measures of the proportionality principle, vote-seat disproportionality (Gallagher, 1991), or theoretically closely related to Duverger's "mechanical effect of electoral systems", the effective number of parties in parliament (Laakso and Taagepera, 1979). We do so in a fashion that allows us to separately evaluate the contribution made by our new measure by incorporating the various special features of elections into it such as legal threshold, multiple tiers, and PR formulas. We also compare the fit of our measure of *effective district magnitude* with that of other approaches to operationalize electoral systems effects. We extend this analysis to a two-stage-model of electoral system effect (appendix C), where we investigate first Duverger's "psychological effect", affecting the effective number of elective parties, and second, how the number of elective parties and institutional rules jointly affect the allocation of seats in parliament ("mechanical effect").

4.1. Data

We examine a wide variety of electoral rules in national parliamentary elections taking place between 1990 and 2014 in 94 democracies worldwide, from all continents. Our sample covers on average 5–6 elections per country. Except for countries with short periods as electoral democracies (we only include elections in years with a Policy IV score of 6 or higher[14]), we cover a minimum of three elections per country, dependent on the availability of comparable data on election results at the national level (see list of sources in appendix B1, and list of elections in appendix B2). The electoral rules range from simple, district-based electoral systems (with clearly defined district magnitude) to more complex rules involving special features such as tiers and thresholds. Later we distinguish presidential regimes (including semi-presidential ones) from parliamentary regimes.[15] Relying on diverse sources, we have gathered information on all the variables we need to measure the *effective district magnitude* according to the rules set up in the theoretical section of this paper,[16] and we have added data on socio-economic diversity measures that have been used previously to help explain electoral system effects. In particular, we use an index of fractionalisation based on the ethnic groups identified by Vogt et al. (2015), and the population size of the countries under study. Our key dependent variables are calculated from electoral outcomes (vote and seat distributions), and taken from the Democracy Barometer (Merkel et al., 2018) and from CLEA (Kollman et al., 2014).[17] Vice-versa, we do not include variables that are themselves features of the party system, such as the number of presidential candidates (but see appendix C3 and C4).

4.2. Regression models

Below, we report a series of regressions with proportionality of outcomes and effective number of parliamentary parties, respectively, as their dependent variable. In this series of regressions, we move step by step, replacing the simple definition of district magnitude with our definition of *effective district magnitude* for one additional electoral rule. Thus, for this data set, we can see how much improvement we see when we better represent the effects of any given special electoral system feature. Furthermore, by including several rules at once, we also take the interactions between multiple features of electoral systems into consideration. This is essential, as most of the more complex rules combine more than one of the features highlighted in this paper, e.g. 84% of all electoral systems with a more proportional PR formula (Sainte-Laguë or largest-reminder), also involve legal thresholds or multiple tiers (see appendix D for an extended analysis of configurations).

The other key feature of our regressions is that we compare the fit of

[12] We lack information on the magnitude of individual districts for the entire set of elections covered in our empirical validity tests (see next section), but we offer a limited validity tests of this approach in appendix A5. The results of this limited test indicate that our geometric mean might account well for the effects that uneven district magnitude might have for the proportionality principle in electoral systems.

[13] Following Adock and Collier (2001, p. 537).

[14] We also include Bosnia and Herzegovina, starting from its first post-war elections in 1996. This country was not rated by Polity due to its partial lack of sovereignty, but its election quality is similar to that of many other countries in our sample.

[15] These are identified using the DPI data (Cruz et al., 2016).

[16] There are some significant coding differences with Bormann and Golder (2013): first, we consider legal electoral thresholds. Second, following our rule 4, we distinguish between MMM/MMP systems based on the existence of a compensatory upper tier, whereas Bormann & Golder's correspondent category is considerably larger, including manifold types of linked mixed systems, e.g., PR rules with bonus upper-tier seats. Similarly, following rule 1 (threshold of exclusion), we consider Taiwan's system with a SNTV lower tier as multi-tier proportional, whereas it features as MMM in Bormann/Golder. We count upper-tier seats in MMM, MMP or multi-tier compensatory PR systems, where the number of seats in the upper tier is flexible (Austria, Germany, Hungary, Romania), strictly according to the rules stipulated in appendix A3; Bormann & Golder code some of them as 'other' (Romania), some according to the seat numbers established by the electoral law (Austria), and others according to the de-facto outcome (Germany).

[17] Except for a few elections in Italy and in Chile (where differences emerge depending on whether calculations are done at the level of parties or electoral alliances), our index of disproportionality correlates highly ($r = 0.96$) with Gallagher's data (2017). In those few cases where we lacked the raw data to make our own calculations, we used the values from Gallagher.

our metric with measures offered in prominent earlier research. Clark and Golder (2006) use district magnitude in the lower tier as their measure of district magnitude, while Li and Shugart (2016), as well as Taagepera and Shugart (2017) rely on the logged MS-product, (lower-tier) district magnitude (M) multiplied by the size of parliament (S).[18] All our multivariate models include robust standard errors (clustered at the level of countries).

4.3. Explaining vote-seat disproportionality

We do not expect that district magnitude perfectly explains disproportionality (or the effective number of political parties). We expect that there will be both mechanical and strategic effects of electoral systems on disproportionality. Strategic effects may confound expectations about mechanical effects. For example, strategic behaviour may reduce support for small parties unlikely to gain seats, and thus reduce both disproportionality and the effective number of parties (see appendix C3 for models that disentangle the two stages of this effect).

Table 3 displays the main results of our estimation. Table 3 restricts these analyses to countries with parliamentary forms of government. We expect that the absence of contamination due to presidential elections will generally lead to an improvement in the R^2 value, and by and large, this is what we find.[19]

District magnitude is consistently negatively correlated with disproportionality. More important for our purpose, however, are the changes in fit (r^2), as we sequentially introduce our four rules into our variable measuring (effective) district magnitude. We start from a measure of lower-tier district magnitude, not incorporating any of our rules (specification 1 & 7). This measure is identical to Clark and Golder's (2006). Building on this basic measure, we introduce our four rules one-by-one, in specifications 2–5 and 8–11. Specifications 2 and 8 are identical to Lublin's (2017) "exclusion magnitude". Each of the four rules improves the empirical fit. The four rules jointly double the R^2 from 20% to 42% (all elections), or from 25% to 52% (parliamentary systems only). The fifth and 11th specification is our universal index, which we label *effective district magnitude*. It considers all complexities of electoral systems that are dealt with in this paper and has the best explanatory power.[20]

Specifications 6 and 12 replicate Taagepera & Shugart's model (2017, p. 139–146), which they label 'MS-product'. We extend it to multi-tier electoral systems, and therefore add a separate variable for the share of upper-tier seats, in line with Taagepera and Shugart's (2017, p. 263–267) 'extended MS-product'. In terms of correlation with disproportionality, it does as well as the lower-tier district magnitude approach of Clark and Golder (2006).

4.4. Explaining the number of parties in parliament

Our second dependent variable is the effective number of parties in parliament. Drawing on previous literature (Ordeshook and Shvetsova, 1994; Amorim Neto and Cox, 1997), we add two proxies for socio-economic diversity to the district magnitude variables in our regression: ethnic fractionalisation and the population size of the country (both logarithmized) (see appendix C4 for interaction models).

We report the same six operationalizations as for the previous dependent variable, starting from a basic model measuring district magnitude in the lower tier (Table 4, specifications 1 and 7), and add features stepwise that satisfy our axioms for dealing with complex electoral system features. In all operationalizations, district magnitude is positively related to the effective number of parliamentary parties. However, its explanatory power rises from 28% (basic model, specification 1) to 36% (specifications 4 & 5), as we move to more sophisticated measures that add further features of electoral systems to our calculation of *effective district magnitude*. In particular, our rule related to legal electoral thresholds (rule 2), and that related to mixed electoral systems (rule 3), improve our understanding of the effective number of parties in parliament.

We obtain similar findings for the subsample of parliamentary systems (Table 4, specifications 7–12),[21] but limiting our sample to parliamentary systems also improves fit, as there are no spill-over effects from presidential elections, and we reach an R^2 of 49%–50% for our fully specified index (specifications 10–11).[22] Both control variables for social diversity show consistently positive effects (except for population size in specifications 6 & 12, as it closely correlates with the number of seats in parliament, one of the elements of the MS-product).

At first sight, however, it might appear that the explanatory power of our models for the number of parties is lower than what is found in earlier work. For example, Clark and Golder (2006, p. 692) reported a very high r^2 of 85%–88%. This is primarily due to the inclusion of the effective number of electoral parties as an explanatory variable (in interaction with institutional explanations). In our data, the number of electoral parties correlates with $r = 0.89$ with the dependent variable; i.e., an empty model (including only the number of electoral parties as explanatory variable) leads to an explanatory power of $r^2 = 79\%$. Adding further variables from Clark & Golder's model and interaction terms increases explanatory power by a one-digit percentage rate, though with $r^2 = 79\%$ in the base model, the differences between alternative operationalisations are small.[23]

Likewise, we have analysed our new model in comparison to Li & Shugart's data (2016, table 4). Many of the differences in the explanatory power are driven by the samples: the R^2 becomes much higher in samples with fewer MMM systems and fewer runoff rules when small islands are included; the latter speaks for models that control for assembly size. Results reported in appendix C show that our approach improves on earlier work, when assessed using the same sample.

We further note that these complex features of electoral systems do not work in isolation, but instead interplay. For instance, most MMP systems include legal thresholds in the upper-tier. Our more extensive analysis of these configurations of the four rules (reported in appendix D, tables D1, D3) shows that our four rules work best if applied as a package – ideally three rules (2, 3, 4) or all four rules jointly.

4.5. Further tests: psychological effect, social cleavages, and the mediating effect of institutions

We undertake further tests with the new index, scrutinising a multi-step model of electoral system effects (Amorim Neto and Cox, 1997; Clark and Golder, 2006), where the mechanism is split into Duverger's "psychological" and "mechanical effect" (appendix C3). In the first

[18] However, in the regressions reported by each set of authors, other variables are added, such as the percentage of parliamentary seats found in an upper tier.

[19] Presidential elections may reduce Duverger's psychological effect vis-a-vis the legislative electoral system (Amorim Neto and Cox, 1997; Clark and Golder, 2006; Li and Shugart, 2016).

[20] Our results also corroborate earlier research: e.g., Farrell's (2011, p. 161) observation that more proportional formulas (Hare, Droop, Sainte-Laguë) produce lower disproportionality than D'Hondt formulas. This is reflected in our models 2 & 8, which show that the PR formulas do matter.

[21] The ambiguous effect of the PR formula – improving the explanation of disproportionality indices, but not of the number of parties in parliament – may be related to the fact that in 74% of the elections where such formula are used, also legal thresholds apply, mitigating the effect of the PR formula on the number of parties.

[22] We exclude South Africa from the analysis, because in spite of its extremely proportional electoral system, its very small party system heavily drives the results.

[23] If we include South Africa, the effect of ethnic fractionalisation varies between different specifications.

Table 3
Explaining vote-seat disproportionality, different indices of district magnitude.

	(1)	(2)	(3)	(4)	(5)	(6)
	Simple	Partial solutions (Rules 1–3)			"Effective m."	"mS product"
a) Specifications 1–6: all elections						
[a] elements considered	Lower-tier m. (Clark & Golder) b/se	PR formula b/se	PR formula + legal threshold b/se	PR formula, legal threshold + multi-tier b/se	PR formula, legal threshold, multi-tier + MMP b/se	Lower-tier m[a] S (Taagepera & Shugart) b/se
(effective) district magnitude (ln)[a]	−0.194** (0.038)	−0.170** (0.031)	−0.246** (0.033)	−0.295** (0.035)	−0.308** (0.036)	
MS-product (ln)						−0.166** (0.029)
upper tier seat share						−0.734* (0.364)
constant	2.205** (0.078)	2.231** (0.080)	2.323** (0.084)	2.460** (0.093)	2.513** (0.099)	3.038** (0.198)
N	516	516	516	516	516	516
R^2	0.201	0.213	0.294	0.387	0.42	0.193

b) Specifications 7–12: only parliamentary elections	(7)	(8)	(9)	(10)	(11)	(12)
	Simple	Partial solutions (Rules 1–3)			"Effective m."	"mS product"
[a] elements considered	Lower-tier m. (Clark & Golder) b/se	PR formula b/se	PR formula + legal threshold b/se	PR formula, legal threshold + multi-tier b/se	PR formula, legal threshold, multi-tier + MMP b/se	Lower-tier m[a] S (Taagepera & Shugart) b/se
(effective) district magnitude (ln)[a]	−0.220** (0.041)	−0.192** (0.034)	−0.281** (0.034)	−0.326** (0.027)	−0.339** (0.027)	
MS-product (ln)						−0.182** (0.030)
upper tier seat share						−1.148* (0.473)
constant	2.282** (0.088)	2.306** (0.089)	2.412** (0.087)	2.553** (0.076)	2.612** (0.076)	3.188** (0.203)
N	364	364	364	364	364	364
R^2	0.255	0.267	0.369	0.477	0.517	0.263

Note: Errors clustered at the country level, robust standard errors in parentheses: + $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. MMP elections with unusual strategic voting (Italy, 2001; Albania, 2005) excluded.

[a] Operationalisation of (effective) district magnitude by models: (1) lower-tier district magnitude, (2) with rule 1 (PR formula), (3) rules 1 & 2 (… & legal thresholds), (4) rules 1, 2 & 3 (… & multi-tier electoral systems), (5) rules 1–4 (… & MMP systems), (6) Taagepera-Shugart district magnitude-seat product (MS) and upper-tier seat share.

Table 4
Explaining the effective number of parties in parliament, by seats (all elections).

	(1)	(2)	(3)	(4)	(5)	(6)
	Simple	Partial solutions (Rules 1–3)			"Effective m."	"mS product"
a) Specifications 1–6: all elections						
[a] elements considered	Lower-tier m. (Clark & Golder) b/se	PR formula b/se	PR formula + legal threshold b/se	PR formula, legal threshold + multi-tier b/se	PR formula, legal threshold, multi-tier + MMP b/se	Lower-tier m[a] S (Taagepera & Shugart) b/se
(effective) district magnitude (ln)[a]	0.130** (0.022)	0.115** (0.018)	0.157** (0.020)	0.179** (0.021)	0.177** (0.021)	
population (ln)	0.053** (0.017)	0.054** (0.017)	0.056** (0.016)	0.051** (0.014)	0.048** (0.014)	−0.003 (0.021)
ethnic fragmentation (ln)	0.145 ([a]) (0.073)	0.147* (0.073)	0.166* (0.070)	0.192** (0.071)	0.190* (0.076)	0.194* (0.083)
MS-product (ln)						0.128** (0.021)
upper tier seat share						0.500** (0.190)
constant	0.103 (0.260)	0.073 (0.253)	−0.014 (0.232)	−0.024 (0.208)	0.018 (0.205)	0.297 (0.243)
N	513	513	513	513	513	513
R^2	0.278	0.293	0.34	0.364	0.358	0.285

b) Specifications 7–12: only parliamentary elections	(7)	(8)	(9)	(10)	(11)	(12)
	Simple	Partial solutions (Rules 1–3)			"Effective m."	"mS product"
[a] elements considered	Lower-tier m. (Clark & Golder) b/se	PR formula b/se	PR formula + legal threshold b/se	PR formula, legal threshold + multi-tier b/se	PR formula, legal threshold, multi-tier + MMP b/se	Lower-tier m[a] S (Taagepera & Shugart) b/se
(effective) district magnitude (ln)[a]	0.134** (0.019)	0.119** (0.016)	0.167** (0.017)	0.189** (0.016)	0.187** (0.016)	
population (ln)	0.057** (0.015)	0.058** (0.015)	0.060** (0.014)	0.054** (0.013)	0.047** (0.013)	−0.004 (0.019)
ethnic fragmentation (ln)	0.158 (*) (0.090)	0.156 (*) (0.089)	0.174* (0.085)	0.211* (0.084)	0.227* (0.087)	0.226* (0.103)
MS-product (ln)						0.132** (0.019)
upper tier seat share						0.494** (0.170)
constant	0.027 (0.221)	0.001 (0.217)	−0.084 (0.205)	−0.076 (0.186)	0.005 (0.184)	0.27 (0.213)
N	352	352	352	352	352	352
R^2	0.37	0.388	0.45	0.496	0.491	0.27

Note: Errors clustered at the country level, robust standard errors in parentheses: + $p < 0.10$, *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$. South Africa excluded.

[a] Operationalisation of (effective) district magnitude by models: (1) lower-tier district magnitude, (2) with rule 1 (PR formula), (3) rules 1 & 2 (… & legal thresholds), (4) rules 1, 2 & 3 (… & multi-tier electoral systems), (5) rules 1–4 (… & MMP systems), (6) Taagepera-Shugart district magnitude-seat product (MS) and upper-tier seat share.

stage, which treats the psychological effect, we assess the contribution of our index to explaining the effective number of elective parties, i.e. the parties winning votes in parliamentary elections, either directly (table C3), or in interactions with social cleavages (table C8). In the second stage, we investigate the mechanical effect, testing how the effective number of electoral parties in interaction with the electoral system variable ([effective] district magnitude) affects vote-seat disproportionality, as well as the number of parties in parliament (tables C4, C5). The theoretical logic on which our index builds orients towards the mechanical effect, and while these empirical findings show that our new index generally is also suitable for testing the psychological effect, we also notice relevant deviations for mixed electoral systems, where actors' strategies seem to be more specific, and not perfectly correlated with the degree of permissiveness of the electoral system. This comes as no surprise, as these deviations, associated with 'contamination effects' under mixed electoral systems, have been well documented in the literature (Herron and Nishikawa, 2001; Cox and Schoppa, 2002). While our new index generally improves the explanatory power of the models, this effect is minor in models where the set of control variables almost perfectly explains the outcome.

5. Discussion

Electoral system proportionality is a central variable in myriad applications in comparative politics, political economy and conflict studies. However, as of the 1990s, most contemporary democracies combine multiple constraints in their electoral systems, which affects the degree to which parties can win proportional representation in parliament, and this complicates the classification of electoral institutions in terms of their "permissiveness". 38 out of 94 democracies have complexified their rules with an upper tier or seat bonuses, and 45 have, at least in one election, excluded parties that did not pass a legal threshold from winning seats.

Here we have offered a theoretically grounded approach to assessing the institutional features of electoral systems related to the proportionality principle. We extend district magnitude, which is widely used as the most important institutional variable related to proportionality, to the more complex electoral rules that are the hallmark of contemporary politics. Building on four axioms, we develop a new single-variable indicator for the institutional side of the proportionality equation which we call the *effective district magnitude*.[24] For 530 elections in 94 democracies worldwide (all values are listed supplementary materials), we showed that our new index correlates well with variables associated with the proportionality of an electoral system by its outcomes, such as disproportionality of the vote-seat allocation and effective number of political parties. The most important improvement comes from the consideration of legal thresholds, on the one hand, and accounting for multiple tiers of seat allocation (e.g., in mixed electoral systems), on the other. However, multiple features of electoral systems often interplay, e.g. legal thresholds might alter the proportionalising effects of MMP systems. This is why the effects of electoral system elements on proportionality are best analysed jointly, rather than in isolation. This comprehensive approach also allows for a comparison of the expected impact of electoral reforms, on a strong theoretical basis.

As Adock and Collier (2001, p. 531) have argued, an empirical variable gains its meaning in relation to a concept. In the present case, the concept and scope is threefold. First, this paper covers Duverger's "mechanical effect" of electoral systems, i.e., the institutional openness of electoral systems to represent parties according to their votes. This means that it does not cover strategic behaviour under electoral systems (the "psychological effect"), which explains why the same electoral systems can lead to different outcomes in terms of proportionality or to a different degree of party concentration, nor does it cover institutional features that shape political behaviour (e.g. the ballot format). Second, our approach builds on the institutional effect of electoral districts, and rules that apply within electoral districts, and is thus suited for the context of nationalised party systems. Extensions of this model to countries with highly territorialised political divides, or to models that consider the number of electoral districts (Taagepera and Shugart, 2017), need to be relegated to future research. Third, there are many further dimensions of democratic representation that are associated with district magnitude (e.g. the personalisation of representation, the representation of social minorities), which are not related to proportionality. For such questions, the complexities of contemporary electoral systems might require a different solution.

[24] Our approach may be expanded to bicameralism if the different parliamentary assemblies have similar party systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper ("How proportional are electoral systems? A universal measure of electoral rules", accepted for publication in Electoral Studies).

Data availability

Data is available from: www.bochsler.eu/data.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.electstud.2023.102713.

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